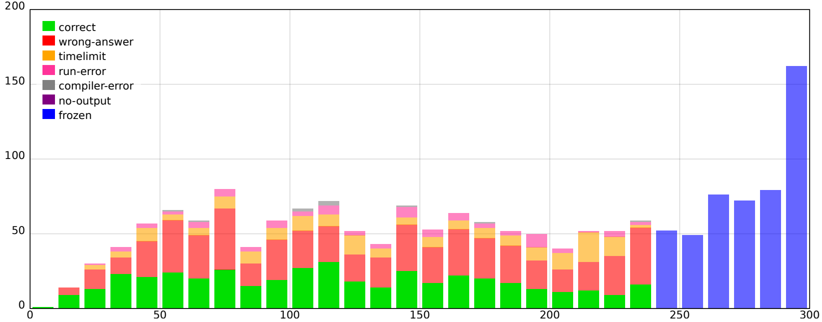
GCPC 2018
Presentation of solutions

acm International Collegiate Programming Contest

FAU FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

Statistics



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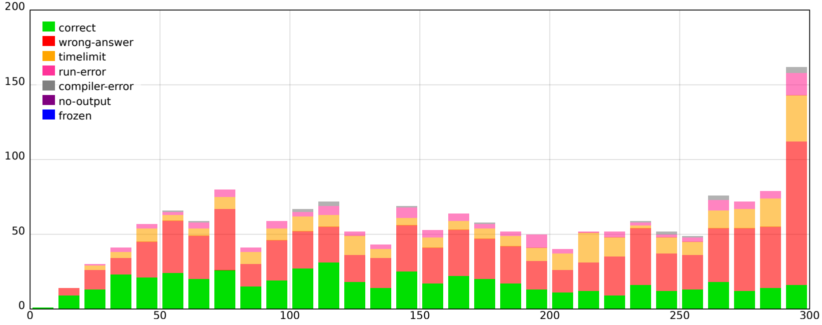

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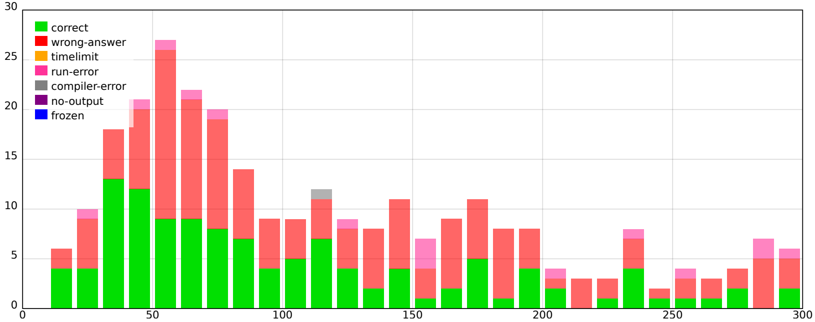
I – It's Time for a Montage



I – It's Time for a Montage

Problem

Given two integer sequences H and V of equal length, find the minimal non-negative d to add to the values in H such that *lexicographically*, $H + d \geq V$.

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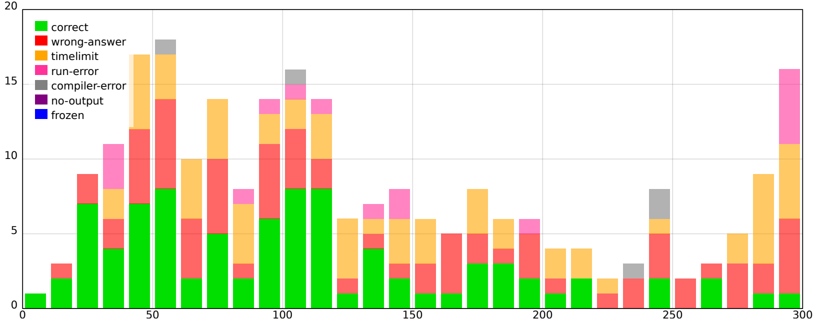
Solution

- If $H_1 > V_1$, the answer is 0.
- Otherwise, put $d := V_1 - H_1$.
- If $H + d < V$, the answer is $d + 1$, else it is d.

C – Coolest Ski Route



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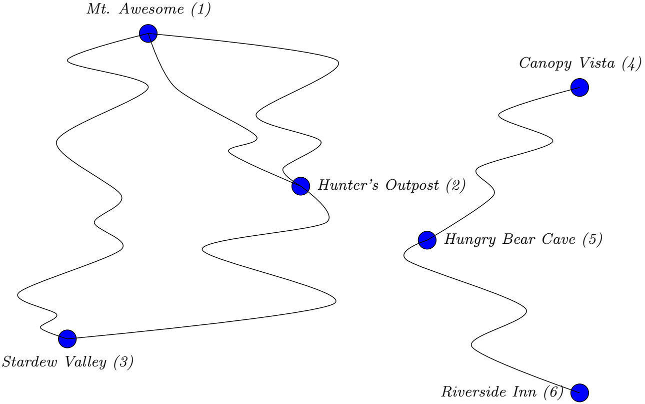
Given a graph of ski slopes labelled with *condition measures*, find a path with maximal sum of these measures.



C – Coolest Ski Route

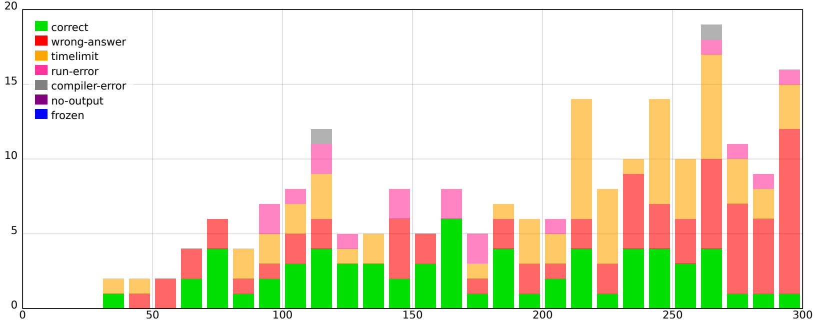
Problem

Given a graph of ski slopes labelled with *condition measures*, find a path with maximal sum of these measures.

Solution

- The graph of ski slopes is a *directed acyclic graph*.
- Find the *longest path* between any two nodes.
- Multiple graph algorithms can be used for this:
  - Invert the measures, use Floyd-Warshall ($\mathcal{O}(n^3)$).
  - Add a super source, invert the measures, then use Bellman-Ford for shortest paths ($\mathcal{O}(n * m)$).
  - Find a topological ordering, then find the maximum for each node with dynamic programming ($\mathcal{O}(n + m)$).

F – Fighting Monsters



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These are Fibonacci numbers!

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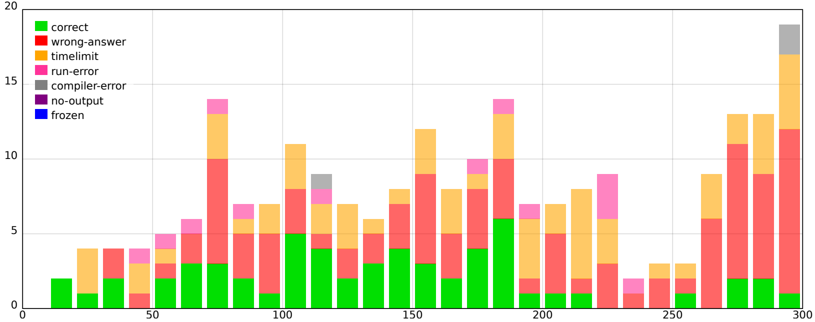
Solution

- Generate Fibonacci numbers up to 10^6.
- Check if two consecutive Fibonacci numbers appear in the list.
- Be careful about monsters with power 1.

D – Down the Pyramid



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Given a non-negative integer sequence A, find the number of possible non-negative integer sequences B such that $A_i = B_i + B_{i+1}$ for all pairs of adjacent numbers in B.

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- B_1 uniquely determines the rest of the sequence.
- If B_1 increases, each odd-numbered element increases and each even-numbered element decreases.

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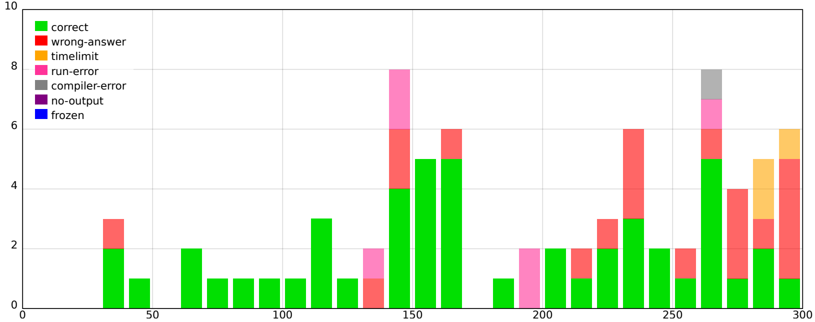
- B_1 uniquely determines the rest of the sequence.
- If B_1 increases, each odd-numbered element increases and each even-numbered element decreases.

Solution

- Put $B_1 := 0$ and compute the resulting sequence.
- Find the minimum odd- and even-numbered elements.
- The range of answers is $[-\min\{B_{2i+1}\}, \min\{B_{2i}\}]$.
- Time complexity: $\mathcal{O}(n)$.

L – Logic Puzzle



L – Logic Puzzle

Problem

Given a rectangular grid of numbers, color some of the cells in black such that the number in each cell equals the number of adjacent black cells. Cells on the boundary may not be colored.

1	1	2	1	1
1	2	3	2	1
1	2	3	2	1
0	1	1	1	0

1	1	2	1	1
1	2	3	2	1
1	2	3	2	1
0	1	1	1	0

L – Logic Puzzle

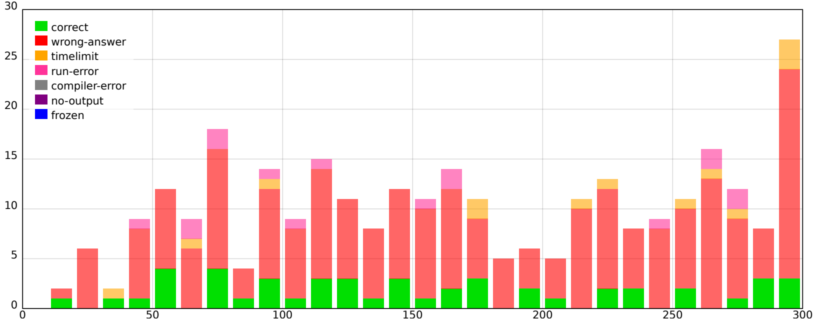
Problem

Given a rectangular grid of numbers, color some of the cells in black such that the number in each cell equals the number of adjacent black cells. Cells on the boundary may not be colored.

Solution

- The solution can be reconstructed row by row.
- For every cell, check if the number to the top left is positive.
- If it is, put an `X` and subtract 1 from every adjacent cell.
- Do a second pass to check if any non-zero numbers remain.
- If there are no conflicts, output the solution.

E – Expired License



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Problem

Given two floating point numbers representing the aspect ratio $a \times b$ of a photo, find prime numbers p and q such that $\frac{a}{b} = \frac{p}{q}$.

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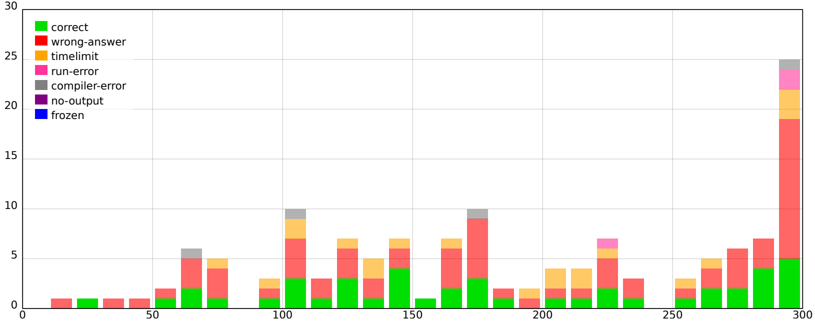
Solution

- Multiply a and b by 10^5 to make both numbers integral.
- Divide both numbers by their greatest common divisor.
- Check whether the resulting two numbers are prime, e.g. using the Sieve of Eratosthenes.
- Special case: quadratic aspect ratios, i.e. 6×6.
- Take care of numerical issues when using floating point arithmetics, e.g. $0.00007 \cdot 10^5 = 6.99999999999999991118\ldots$

H – Hyper Illuminati



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Build an n-dimensional step-pyramid consisting of m blocks.

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Observation

- We need to find a dimension n and a step number s such that

$$\sum_{k=1}^{s} k^{n-1} = m.$$

- The k^{n-1} grow very fast, especially for higher dimensions.

H – Hyper Illuminati

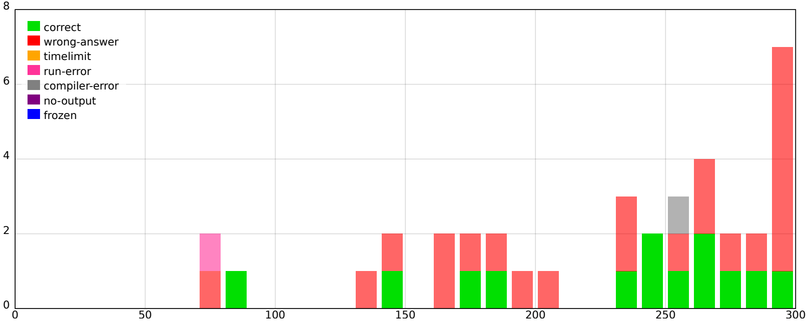
Solution

Calculate all candidate pyramids keeping the following in mind:

- We can stop increasing s as soon as $s^{n-1} > m$.
- We can stop increasing n as soon as $2^{n-1} > m$.

In total, there are only 328 373 pyramids with two or more steps and at most 10^{16} blocks.
Possible pitfall: avoid the `pow()` function in C++ and Java.

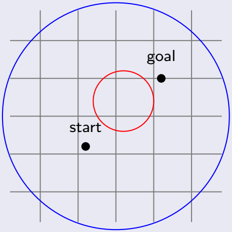
B – Battle Royale



B – Battle Royale

Problem

Given a **red circle**, a **blue circle** and **two points**, find the length of the shortest path between the two points while staying inside the blue circle and outside the red circle.

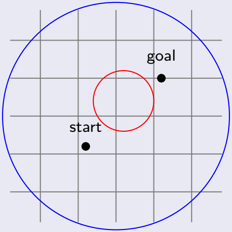


Simplifications:

- Touching the circles is allowed.
- The direct path is always blocked by the red circle.
- The red circle and the two points are completely inside the blue circle.

B – Battle Royale

Problem

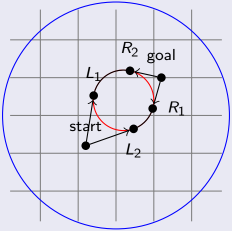


Deductions

- The blue circle is irrelevant.
- No need two check if the direct connection is possible.

B – Battle Royale

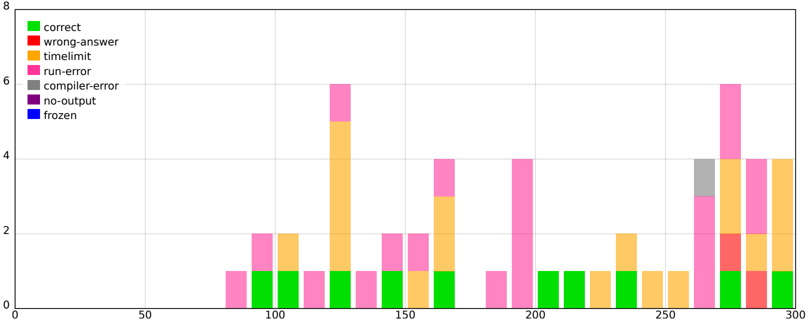
Problem



Solution

- Compute **tangents** of the red circle going through start and end.
- Compute the length of **circle segments** between the touching points $L_{1,2}$, $R_{1,2}$.
- Find the minimum length of the four paths $start \to L_{1,2} \to R_{1,2} \to end$.

A – Attack on Alpha-Zet



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Given a sequence of locations inside a maze, what is the minimal distance one has to travel to visit them all in given order?

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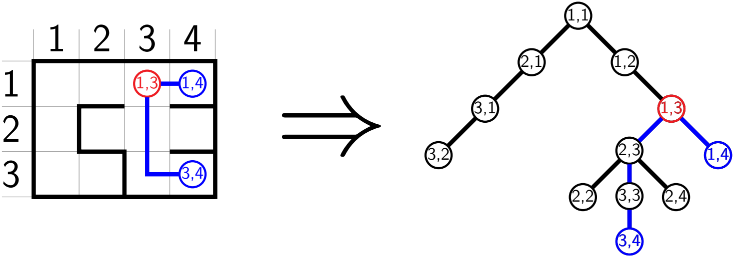
Observation

- The number of locations is too big for multiple iterations of any search algorithm.
- Since the maze has no loops, it can be seen as a tree.

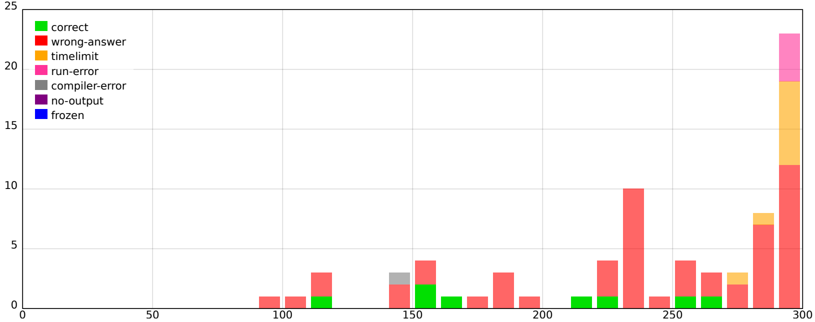
A – Attack on Alpha-Zet



Solution

- Transform the maze into a tree e.g. with *depth first search*.
- Find *Lowest Common Ancestors* to calculate the distance between two consecutive locations.
- $\Rightarrow$ Init: $\mathcal{O}(n \log n)$, Lookup: $\mathcal{O}(1)$

K – Kitchen Cable Chaos



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Given n cables with lengths $d_1, \ldots, d_n$, find the set of cables that has the *largest minimal overlap* if fitted in the gap g.

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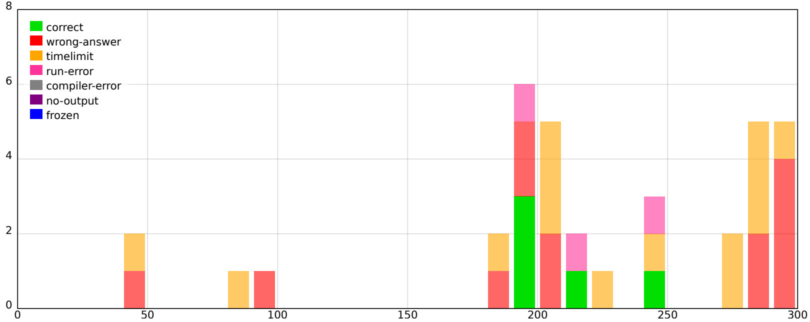
Solution

- Calculate all pairs (i, j) such that there is a set of i cables with total length j.
- This can be done with a knapsack-like DP in $\mathcal{O}(n^2 \cdot g)$.
- The largest minimal overlap for a pair (i, j) is given by $\frac{j+10-g}{i+1}$.
- Try all pairs to find the best solution.

M – Mountaineers



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Problem

Given are some pairs of points in a topographic map. For each pair find the least maximal height of a path connecting both ends.

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Intuition

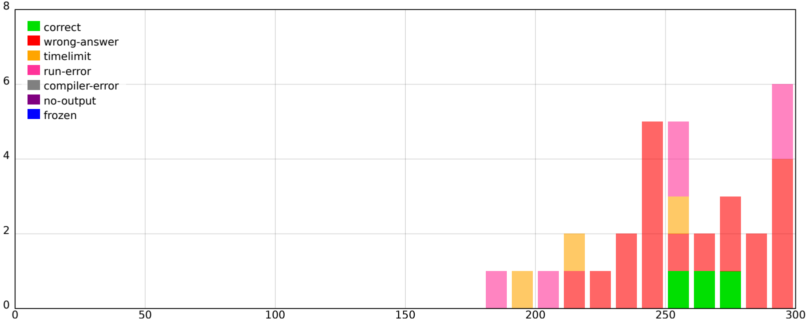
- Imagine a rising water line throughout the mountain range.
- The answer for a pair is the lowest height at which it becomes possible to swim from one end to the other.

M – Mountaineers

Solution

- Store connected components in a union-find data structure. In each component, store a list of end points.
- Merge neighboring cells by increasing height.
- While merging:
  - Always merge the smaller list into the larger list.
  - If both ends of a pair are in the two lists, the current height is the answer for that pair.
- Total time complexity: $\mathcal{O}(k \cdot \log^2 k)$ where $k = \max(m \cdot n, q)$.

J – Jigsaw Puzzle



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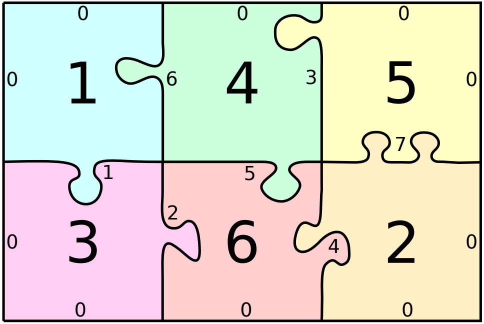
Assemble a jigsaw puzzle from square shaped pieces with jagged edges.



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Assemble a jigsaw puzzle from square shaped pieces with jagged edges.

Solution 1

- Pick an arbitrary piece and place it at $(0, 0)$.
- Run a breadth-first-search from this piece, fixing coordinates and orientation of the other pieces along the way.
- Check that this is a valid solution (no gaps/overlaps/...).
- Each step can be done in time $\mathcal{O}(n)$.

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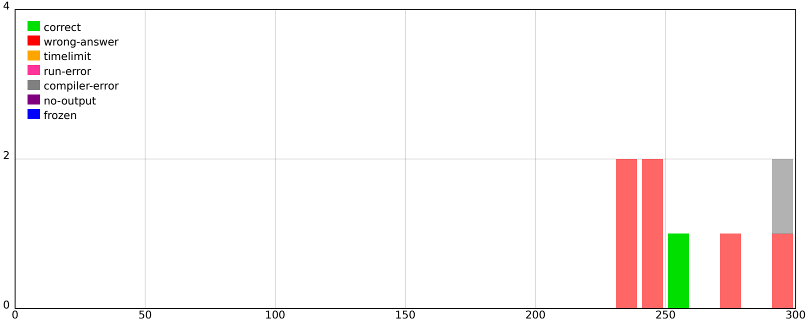
Solution 2

Find a corner piece and reconstruct the solution row by row.
Be *very* careful when checking the connections.

G – GPS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Given a point $\vec{x}$ on a sphere of radius r and a point $\vec{y}$ outside the sphere, compute whether the line $\overline{\vec{x}\vec{y}}$ intersects the sphere and if not, output $|\overline{\vec{x}\vec{y}}|$.

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Solution, part II

Assume $\vec{x}$ and $\vec{y}$ are given in cartesian coordinates (x, y, z).
To check whether the line $\overline{\vec{x}\vec{y}}$ intersects the sphere:

- Compute closest point $\vec{p}$ on the line to $\vec{0} = (0, 0, 0)$ (vector projection, "Lot fällen"). You can use your 2D-Formula for this. It is also correct in higher dimensions.
- Check whether $|\vec{p}|$ is larger than r. If not, the line will intersect the sphere.

Alternatively, check on which side of the tangential plane $\vec{y}$ lies (normal vector of the plane is $\vec{x}$) using dot product.

Solution, part I

Points are not given in cartesian coordinates ...

G – GPS

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1. $\vec{x}$: Given in terms of geographic coordinates (ℓ_o, ℓ_a). These are (similar to) polar coordinates – their conversion is known.

G – GPS

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1. $\vec{x}$: Given in terms of geographic coordinates (ℓ_o, ℓ_a). These are (similar to) polar coordinates – their conversion is known.

2. $\vec{y}$: Circle that intersects the XY-plane at (at least) two points with an angle ψ. One of the points is at longitude ϕ. The point is where one has covered $x\%$ of the circle from ϕ.

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Compute the coordinate stepwise:

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Compute the coordinate stepwise:

- Start with circle in equatorial plane of correct radius. Find point on $x\%$ of the circle.

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